Measurement-induced quantum operations on multiphoton states

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We investigate how multiphoton quantum states obtained through optical parametric amplification can be manipulated by performing a measurement on a small portion of the output light field. We study in detail how the macroqubit features are modified by varying the amount of extracted information and the strategy adopted at the final measurement stage. At last the obtained results are employed to investigate the possibility of performing a microscopic-macroscopic non-locality test free from auxiliary assumptions.

I. INTRODUCTION

The possibility of performing quantum operations in order to tailor quantum states of light on demand has been widely investigated in the last few years. Several fields of research have been found to benefit from the capability of generating fields possessing the desired quantum properties. Non-classical states of light, such as sub-Poissonian light [1], squeezed light [2, 3] or the quantum superposition of coherent states [4, 5], have been generated in a conditional fashion. In this context, continuous-variable (CV) quantum information represents one of the most promising fields where conditional and measurement-induced non-Gaussian operations can find application. To this end, quantum interactions can be induced by exploiting linear optics, detection processes and ancillary states [6]. For example, the process of coherent photon-subtraction has been exploited to increase the entanglement present in Gaussian states [7, 8] and to engineer quantum operations in travelling light beams [9]. Finally, very recently conditional operations lead to the realization of different schemes for the implementation of the probabilistic noiseless amplifier [10–12], which can find interesting application within the context of quantum phase estimation [13].

Strictly related to the engineering of quantum states of light for applications to quantum information, there is the problem of beating the decoherence due to losses which affect quantum resources interacting with an external environment. In the last few years a large investigation effort has been devoted to the decoherence process and the robustness of increasing size quantum fields, realized by non linear optical methods [14–17]. Recently quantum phenomena generated in the microscopic world and then transferred to the macroscopic one via parametric amplification have been experimentally investigated. In Ref.[16] it has been reported the realization of a resilient to decoherence multiphoton quantum su-

perposition (MQS) [18] involving a large number of photons and obtained by parametric amplification of a single photon belonging to a microscopic entangled pair: $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$, where A, B refer to spatial modes $\mathbf{k_A}, \mathbf{k_B}$ and the kets refer to single photon polarization states $\vec{\pi}$ ($\pi = H, V$). This process has been realized through a non linear crystal pumped by a UV high power beam acting as a parametric amplifier on the single entangled injected photon, i.e. the qubit $|\phi\rangle_B$ on spatial mode $\mathbf{k_B}$. In virtue of the unitarity of the optical parametric amplifier (OPA), the generated state was found to keep the same superposition character and the interfering properties of the injected qubit [14, 15, 19] and, by exploiting the amplification process, the single photon qubit has been converted into a macro-qubit involving a large number of photons.

In this paper we consider several strategies for the realization of measurement-induced quantum operations on these multiphoton states, generated thought the process of optical parametric amplification. We investigate theoretically how the measurement strategies, applied on a part of the multiphoton state before the final identification measurement, affect the distinguishability of orthogonal macro-qubits. Such measurements based on the discrimination of multiphoton probability distributions combine features of both continuous and discrete variables techniques. The interest in improving the capability of identifying the state generated by the quantum injected optical parametric amplifier (QIOPA) system mainly relies in two motivations: the first one concerns the development of a discrimination method able to increase the transmission fidelity of the state after the propagation over a lossy channel, and hence to overcome the imperfections related to the practical implementation. Such increased discrimination capability in lossy conditions could find applications within the quantum communication context. The second reason concerns the scenario in which an appropriate pre-selection of the macro-qubits could be adopted to demonstrate the micro-macro non-locality, free from the auxiliary assumptions requested if the filtering procedure was applied at the final measurement stage.

In previous papers [15, 16] a probabilistic discrimi-

(a)

EPR
source

PBS k_P

$k_{P'}$

k_A

OPA

k_B

UBS

k_B

Shutter

Detection

$U_z = e^{-i\varphi\sigma_z/2}$

Z

|H⟩

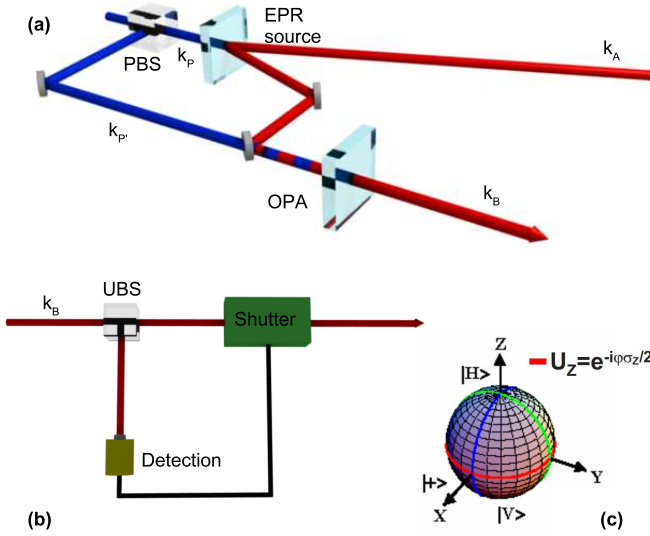|+⟩

Y

X |V⟩

(b)

(c)

FIG. 1: (Color online) (a) Quantum injected optical parametric amplifier (QIOPA) scheme: an entangled photon pair is generated through spontaneous parametric down conversion (SPDC). One of the two photon is amplified by a non linear crystal, realizing the optimal phase covariant cloning of the injected qubit. (b) Scheme of the measurement-induced quantum operation process. The field is split by an unbalanced beam splitter (UBS), and reflected portion is measured to conditionally active the optical shutter placed in the path of the transmitted portion of the field. (c) Schematic view of the single photon Bloch sphere. The QIOPA device performs the optimal phase covariant process, hence amplifying equally all photons belonging to the equator of the sphere (red line), with polarization $\vec{\pi}_\phi = 2^{-1/2}(\vec{\pi}_H + e^{i\phi}\vec{\pi}_V)$.

nation method, the orthogonality-filter (OF), was introduced and successfully applied to an entanglement test in a microscopic-macroscopic bipartite system. The application of the OF strategy, acting at the measurement stage, is indeed not suitable for the demonstration of loophole-free micro-macro non-locality because of the presence of inconclusive results [20]. These correspond to the selection of different sub-ensembles of data, depending on the choice of the measurement basis. In Ref. [21] Popescu showed that the pre-selection of data before the final measurement could encompass the problem of a base dependent filtering of the detected state. More specifically, he showed that performing a sequence of measurements on each of the two parts of a bipartite state could reveal the presence of *hidden* non-locality which was not observable with a single measurement. This method corresponds to a selection of a sub-ensemble of data *independently* from the measurement performed in the non-locality test. This pre-selection scenario was subsequently extended by Pawlowski *et al.* in Ref. [22] in the more general context of quantum communication.

The capability to generate at the output of the parametric amplifier a quantum state of large size allows one to act on a small portion of the field in order to mod-

ify the features of the remaining part by a suitable selection. Conditional manipulation of quantum states of light depending on measurement carried out on part of the beam could increase the capability of discriminating among the generated multiphoton states, as suggested in [23]. Starting from the original proposal of a preselection apparatus in a macro-macro configuration of Ref. [24], we consider the particular case in which a macro-state generated by the QIOPA is split by an unbalanced beam splitter (UBS) and manipulated by measuring the state on the reflected mode. The conceptual scheme underlying the present investigation is shown in figure 1-(b): a part of the wave-function is measured and the results of the measurement are exploited to conditionally activate an optical shutter placed in the transmitted part. Such shutter, whose realization has been recently reported in Ref.[23], is used to allow the transmission of the optical beam only in presence of a trigger event, i.e. in this case the results of the measurement performed in the reflected part of the state. Several detection schemes will be investigated in this paper. In fig.2-I is reported the filtering method based on the intensity pre-selection. As analyzed in more details in section II, the signal of the reflected part of the macro state is analyzed by a threshold detector. If the measured intensity value is above a certain threshold, the shutter on the transmitted mode is activated. This strategy allows to overcome the experimental imperfection related to the vacuum injection into the amplifier, and hence to distillate the macro state from the noise belonging to the crystal spontaneous emission. The application of this scheme has been already proposed by Stobinska *et al.* in Ref. [25] to perform a loophole free nonlocality test in a macroscopic-macroscopic configuration. In fig2-II is reported the strategy illustrated in section III, based on a probabilistic discrimination of the reflected macro-qubit part performed by the OF device. By changing the polarization analysis basis on the reflected mode, we have investigated how the macro-state visibility obtained by a dichotomic measurement of the transmitted state is affected. In fig.2-III is illustrated the measurement procedure described in section IV, in which both the reflected and the transmitted mode are analyzed by a probabilistic OF-based measurement. At last, section V addresses the case in which the reflected macro qubit part is measured in two different polarization basis, as shown in figure 2-IV, and the final measurement is purely dichotomic. This measurement strategy is aimed at the realization of a non-locality test on the micro-macro photon state, without any auxiliary assumption. However, we show that such scheme does not allow to obtain a violation of a Bell's inequality since the analyzed strategy has the effect of increasing the correlations present in the micro-macro system only in a specific polarization basis while suppressing the correlations in the other basis.
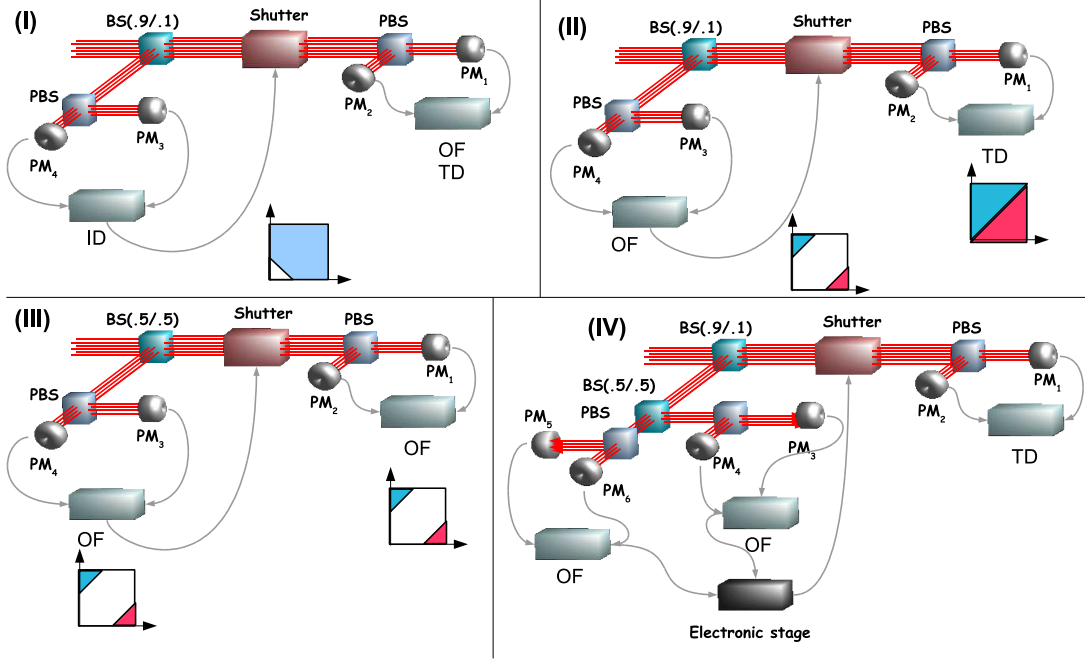
FIG. 2: (Color online) Measurement strategies devoted to the manipulation of the macro-qubit state: (I) the shutter activation is conditioned to an intensity measurement on the reflected portion of the macro state; (II) the small reflected part of the macro state is analyzed in polarization and detected through an OF based measurement strategy; (III) the macro state is split in two equal parts, and both the reflected and the transmitted components are detected through an OF device; (IV) a double basis measurement is performed on the small reflected portion of the macro qubit.

II. FILTERING OF THE MACRO-QUBIT

In this section we discuss the specific preselection scheme, sketched in Fig. 2-(I), proposed in Ref. [25] in a macro-macro scenario. One of the main experimental challenge for the realization of the micro-macro system of Fig.1 is the achievement of spectral, spatial and temporal matching between the optical mode of the injected single photon state and the optical mode of the amplifier. In ideal conditions, as shown in [15, 16], the micro macroscopic system is realized by the amplification process performed over an entangled couple $|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$ generated by the EPR source in figure 1-(a). In realistic conditions, the injected micro-micro system is given by: $\hat{\rho}_{\psi^-} = p|\psi^-\rangle_{AB}\langle\psi^-| + (1-p)/2\hat{I}_A \otimes |0\rangle_B\langle 0|$, where, as said, $|\psi^-\rangle_{AB}$ is the entangled singlet state connecting the spatial modes A and B, and the parameter p expresses the amount of mode-matching between the seed and the amplifier. The micro-macro amplified state $\hat{\rho}_{\Psi^-}$ is obtained after the amplification of the $\hat{\rho}_{\psi^-}$ injected state: $\hat{\rho}_{\Psi^-} = (\hat{I}_A \otimes \hat{U}_B)\hat{\rho}_{\psi^-}(\hat{I}_A \otimes \hat{U}_B^\dagger)$, where $\hat{U} = e^{-\imath\hat{H}_I t/\hbar}$ is the time evolution operator associated with the amplifier, defined by the interaction Hamiltonian $\hat{H}_I = \imath\hbar\chi\hat{a}_H^\dagger\hat{a}_V^\dagger + \text{H.c.}$. Then in the expression of the number of photons $N_{\pi_\pm}(\varphi)$ generated by the amplifier when a single photon with equatorial [Fig.1-(c)] polarization state $|\phi\rangle$ is injected, the spontaneous emission has to be taken into account:

$N_{\pi_\pm}(\varphi) = p[\bar{m} + \frac{1}{2}(2\bar{m} + 1)(1 \pm \cos(\varphi))] + (1-p)\bar{m}$. When the single photon is injected correctly in the OPA, a pulse with a higher photon number is generated since stimulated emission processes occur in the amplifier. The filtering method here presented exploits this feature to reduce the noise introduced by the spontaneous emission of the amplifier.

Let us now discuss the propagation of the multiphoton field produced by the amplifier and the pre-selection procedure obtained through an intensity threshold detector (ID) and the shutter device. As shown in fig.2-I, the amplified state is split by an unbalanced beam splitter (UBS) $0.90 - 0.10$ in two parts: the smaller portion on mode $\mathbf{k_D}$ is analyzed by the ID, and the larger one on mode $\mathbf{k_C}$ is conditionally pushed through a polarization preserving shutter [23], and measured in polarization by a dichotomic measurement. The ID based filtering strategy allows then to obtain a better discrimination between the orthogonal macro states, by minimizing the noise related to the vacuum injection into the amplifier. It is worth nothing that, at variance with the techniques which will be introduced in the following sections, the ID action is invariant for rotation on the Fock space. It indeed selects the same region of the macro-qubit either in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis either in the $\{\vec{\pi}_R, \vec{\pi}_L\}$ one. The action of the ID on mode $\mathbf{k}_D$ and of the shutter on mode $\mathbf{k_C}$ allows to distill the macro-qubit from the noise generated by the amplifier and related to the spontaneous emission of the crystal. In the ideal case, this measure-
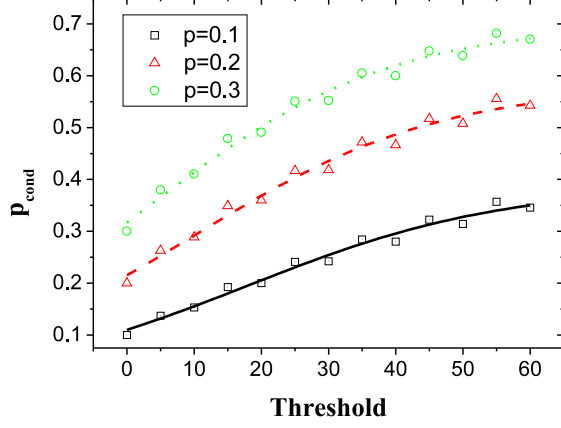
FIG. 3: (Color online) Trend of the injection probability as a function of the ID threshold, for different initial values of p. The nonlinear gain of the amplifier is set at $g = 1.5$.

ment corresponds to the projection of the impinging field onto the subspace: $\hat{\Pi}_k = \sum_{m+n>h} |n\pi, m\pi_\perp\rangle \langle n\pi, m\pi_\perp|$, where $|n\pi, m\pi_\perp\rangle$ represents a quantum state with n photons with polarization π and m photons with polarization $\pi_\perp$. The measurement method is hence based on a threshold detection scheme, in which the ID clicks only if $n_\pi + m_{\pi_\perp} > h$, where h is a threshold conveniently selected. This click activates the shutter on the transmitted UBS mode, ensuring the presence of the higher, i.e. correctly injected, pulses. This scheme has the peculiar property of selecting an invariant region of the Fock space with respect to rotations of the polarization basis. As said, the action of the ID device allows to decrease the noise due to the vacuum injection into the amplifier since it preserves only the higher pulses, and hence the ones that, with an higher probability, belong to the amplification process. These considerations can be quantified in the following way. The parameter of interest is the conditional injection probability, i.e. the injection probability conditioned to the activation of the shutter given by the threshold condition of the ID. We then evaluated numerically this quantity for several values of the unconditioned injection probability p. It turns out that the value of p_{cond} is increased as shown in Fig.3, in which we report the trend of the conditional injection probability p_{cond} as a function of the ID threshold h.

III. DETERMINISTIC TRANSMITTED STATE IDENTIFICATION

In this section we are interested in exploiting the action of a different pre-selection strategy, no more based on the intensity filtering but on a comparison between orthogonally polarized signals. This configuration is illustrated in fig.2-II and is based on a peculiar feature of the equatorial macro states. Indeed, any macro-qubit be-
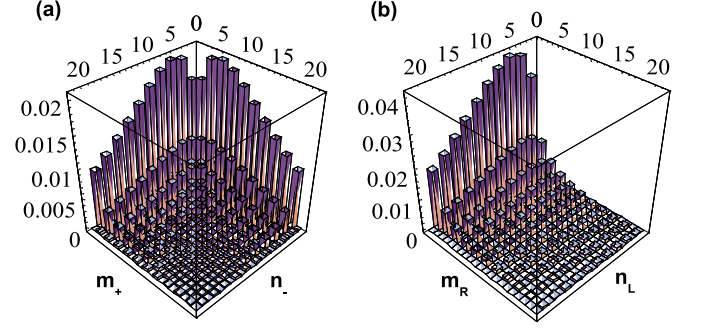

FIG. 4: (Color online) (a) Probability distribution of the state $|\Phi^R\rangle$ as a function of the number of photons $\{\vec{\pi}_+, \vec{\pi}_-\}$. (b) Probability distribution of the state $|\Phi^R\rangle$ as a function of the number of photons $\{\vec{\pi}_R, \vec{\pi}_L\}$. In both distributions $g = 1.5$.
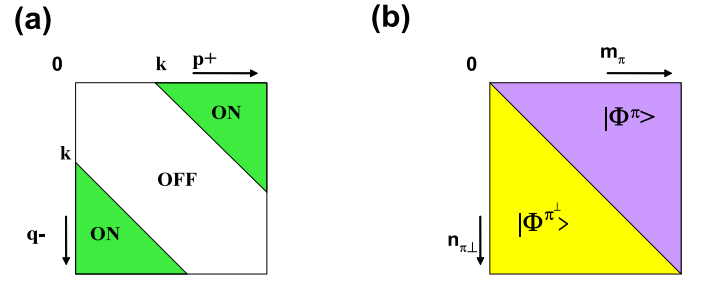

FIG. 5: (Color online) (a) Measurement scheme adopted for the conditional activation of the shutter: if the OF, on the reflected mode, measures the state on the green regions, the shutter, on the transmitted mode, is conditionally activated. The green regions correspond to the state for which the signals belonging to orthogonal polarizations are unbalanced over a certain threshold k, i.e. $|p - q| \geq k$. (b) Scheme for the final detection of the output state: Conditioned on a measurement result in the ON region on the reflected mode, the transmitted mode is identified by a dichotomic measurement in the $\{\pi, \pi_\perp\}$ basis. The diagonal contribution to the quantum state is assigned randomly to the state $|\Phi^\pi\rangle$ or $|\Phi^{\pi\perp}\rangle$.

longing to the injection of an equatorial qubit, due to the phase covariance of the amplifier, can be discriminated through a measurement based on a comparison. Precisely, we can measure the intensity signals belonging to orthogonal polarization components of the same macro-state and subsequently compare them above a certain threshold k. If analyzed in the same polarization basis of the injected qubit, the two signals will be unbalanced with an high probability. This can be explained by analyzing the probability distribution of the amplified states, reported in figure 4: (a) in the mutually unbiased equatorial polarization basis with respect to the injected state and (b) in the same basis as the injected qubit one.

We will address two cases in which the state generated by the amplifier is either $|\Phi^+\rangle$ or $|\Phi^R\rangle$, obtained by the amplification of a single photon polarized $\vec{\pi}_+ = \frac{\vec{\pi}_H + \vec{\pi}_V}{\sqrt{2}}$ and $\vec{\pi}_R = \frac{\vec{\pi}_H + i\vec{\pi}_V}{\sqrt{2}}$, respectively. In both cases the analy-

sis basis corresponding to the UBS reflected mode is fixed to $\{\vec{\pi}_+, \vec{\pi}_-\}$, while the transmitted mode is analyzed in the same basis in which the injected qubit has been encoded. Let us discuss the experimental setup shown in Figure 2-II. The macro-state $|\Phi^+\rangle$ (or $|\Phi^R\rangle$) generated by the QIOPA impinges on the UBS. A small portion of the field is reflected on mode $\mathbf{k_d}$ and measured on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. The two signals belonging to orthogonal polarizations are then compared by an orthogonality filter (OF). When the two signals are unbalanced, i.e. $|p - q| > k$, being p, q the number of photons $\vec{\pi}_+, \vec{\pi}_-$ polarized and k an appropriate threshold, the shutter on mode $\mathbf{k_c}$ is activated and the field on that mode is conditionally transmitted (see Fig.5). The macro-state $|\Phi^+\rangle$ ($|\Phi^R\rangle$) is then analyzed in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ (or $\{\vec{\pi}_R, \vec{\pi}_L\}$) basis. In the following sections we will address the problem of discriminating the final macro-state, given the acquired information on the small portion of the reflected field.
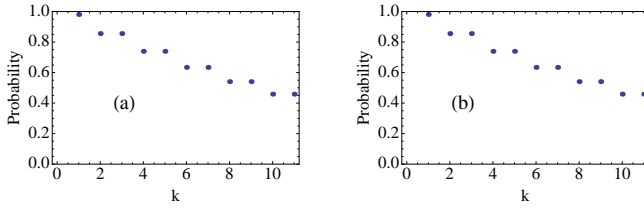

FIG. 6: (Color online) (a) Probability of activating the shutter when the state $|\Phi^R\rangle$ is analyzed in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis versus the threshold k of the OF. (b) Probability of activating the shutter when the state $|\Phi^+\rangle$ is analyzed on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. The nonlinear gain of the amplifier is set at $g = 1.5$.

A. Probability of shutter activation

Let us first evaluate the probability P of activating the shutter when the impinging state is detected on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis, depending on the value of k, with an OF technique. As shown in figure 6, the probability of activating the shutter is the same for the two output fields $|\Phi^+\rangle$ and $|\Phi^R\rangle$. This result can be explained by considering the probability distributions of the state $|\Phi^R\rangle$ in the two mutually unbiased equatorial bases shown in figure 4. Due to the linearity of the quantum mechanics, the state $|\Phi^R\rangle$ can be written as $|\Phi^R\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + \imath|\Phi^-\rangle)$. Hence, due to the peculiar features of the two macro-states $|\Phi^{\pm}\rangle$, that have non-zero contributions for terms with different parity, the probability distribution of the macro-state $|\Phi^R\rangle$ in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis is given as the sum of the two probability distributions of the states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ in the same basis. Since shot by shot the OF identifies the state $|\Phi^+\rangle$ or $|\Phi^-\rangle$ with the same probability, the activation of the shutter has the same probability of occurrence for any linear combination of $|\Phi^-\rangle$ and $|\Phi^+\rangle$ impinging on the BS.

B. Analysis of the Macro-state $|\Phi^+\rangle$

Let us analyze the evolution of the state $|\Phi^+\rangle$ passing through an unbalanced beam-splitter (UBS). We start with the expression of the macro-qubit [16]:

$$|\Phi^+\rangle = \frac{1}{C^2} \sum_{ij}^{\infty} \left(\frac{-\Gamma}{2}\right)^j \left(\frac{\Gamma}{2}\right)^i \frac{\sqrt{(2j)!(2i+1)!}}{j!i!}$$
$$\times |(2i+1)+, 2j-\rangle_b = \qquad (1)$$
$$= \frac{1}{C^2} \sum_{ij}^{\infty} \left(\frac{-\Gamma}{2}\right)^j \left(\frac{\Gamma}{2}\right)^i \frac{(\mathbf{b}_+^\dagger)^{2i+1}}{j!} \frac{(\mathbf{b}_-^\dagger)^{2j}}{i!}|0\rangle$$

The UBS transformation equations for the creation operators on spatial mode $\mathbf{b}$ read:

$$\mathbf{b}_{\pm} = \sqrt{\tau}\mathbf{c}_{\pm} + \imath\sqrt{1-\tau}\,\mathbf{d}_{\pm} \qquad (2)$$

where the subscript $\pm$ refers to the polarization modes $\vec{\pi}_{\pm} = \frac{\vec{\pi}_H \pm \vec{\pi}_V}{\sqrt{2}}$, and $\mathbf{c}^\dagger$ and $\mathbf{d}^\dagger$ refer to the creation operators on the spatial modes transmitted and reflected by the UBS. Hence after the UBS the output state becomes:

$$|\Phi^+\rangle^{out} = \frac{1}{C^2} \sum_{ij}^{\infty} \sum_{k}^{2i+1} \sum_{l}^{2j} \binom{2i+1}{k} \binom{2j}{l} \left(\frac{-\Gamma}{2}\right)^j$$
$$\left(\frac{\Gamma}{2}\right)^i \frac{1}{j!} \frac{1}{i!} \left(\sqrt{\tau}\mathbf{c}_+^\dagger\right)^k \left(\imath\sqrt{1-\tau}\mathbf{d}_+^\dagger\right)^{2i+1-k} \quad (3)$$
$$\left(\sqrt{\tau}\mathbf{c}_-^\dagger\right)^l \left(\imath\sqrt{1-\tau}\mathbf{d}_-^\dagger\right)^{2j-l} |0\rangle$$

By applying the creation operators to the vacuum state the output state reads:

$$|\Phi^+\rangle^{out} = \frac{1}{C^2} \sum_{ij}^{\infty} \sum_{k}^{2i+1} \sum_{l}^{2j} \left(\frac{-\Gamma}{2}\right)^j \left(\frac{\Gamma}{2}\right)^i \frac{1}{j!} \frac{1}{i!} \frac{\sqrt{\tau}^{k+l}}{\sqrt{k!}}$$
$$\frac{(2i+1)!(2j)!}{\sqrt{(2i+1-k)!(2j-l)!}} \frac{(\imath\sqrt{1-\tau})^{2i+1+2j-k-l}}{\sqrt{l!}}$$
$$|k+, l-\rangle_d |(2i+1-k)+, (2j-l)-\rangle_c$$
$$(4)$$

Let us now consider the case in which the reflected mode by the UBS is measured on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. The state $|p+, q-\rangle_d$ is detected on the reflected mode, the transmitted state then reads:

$$|\Phi^+\rangle^{meas} = \frac{1}{C^2} \sum_{ij}^{\infty} \left(\frac{-\Gamma}{2}\right)^j \left(\frac{\Gamma}{2}\right)^i \frac{\sqrt{\tau}^{2i+1+2j-p-q}}{j!}$$
$$\frac{(\imath\sqrt{1-\tau})^{p+q}}{i!} \frac{(2i+1)!(2j)!}{\sqrt{p!q!(2i+1-p)!(2j-q)!}}$$
$$|(2i+1-p)+, (2j-q)-\rangle_c \qquad (5)$$

We are interested in investigating the distinguishability between orthogonal macro-states by varying the pre-selection performed over the multiphoton state itself. It is worth noting that our investigation addresses the coherence between two different polarization modes which is quantified by the visibility of the intensity curve obtained by rotating a polarizer. Our first scope is to investigate the visibility of the transmitted mode as a function of the unbalancement between $\vec{\pi}_+$ and $\vec{\pi}_-$ photons, detected on the reflected mode. Namely, if $|p - q| > k$ on mode $\mathbf{k_d}$, what is the visibility of the state $|\Phi^+\rangle^{meas}$ on mode $\mathbf{k_c}$?

This quantity can be quantified in the following way. Due to the peculiar shape of the photon number probability distribution (figure 4-(b)), the identification of the state $|\Phi^+\rangle$ can be performed by discriminating between the number of photons $\vec{\pi}_+$ and $\vec{\pi}_-$ polarized. Let us define the following quantities: $P^+(m, n|p, q)$ is the probability that, if the state $|p+, q-\rangle_d$ is detected on spatial mode $\mathbf{k_d}$, $m > n$ is obtained on spatial mode $\mathbf{k_c}$, and hence the macro-state $|\Phi^+\rangle$ is identified (m, n being the number of photons $\vec{\pi}_+$ and $\vec{\pi}_-$ polarized). On the contrary $P^-(m, n|p, q)$ is the probability that, given the detection of the state $|p+, q-\rangle_d$ on spatial mode $\mathbf{k_d}$, $n > m$ is obtained on spatial mode $\mathbf{k_c}$, and hence the macro-state $|\Phi^-\rangle$ is identified, even if the initial state impinging on the UBS was $|\Phi^+\rangle$. We can than derive the visibility as a function of the threshold k such that $|p - q| > k$:

$$V(k) = \frac{\sum_{m,n} \sum_{p,q} \left(P_{m,n}^{p,q\,+}(k) - P_{m,n}^{p,q\,-}(k) \right)}{\sum_{m,n} \sum_{p,q} \left(P_{m,n}^{p,q\,+}(k) + P_{m,n}^{p,q\,-}(k) \right)} \quad (6)$$

where $P_{m,n}^{p,q\,\pm}(k) = P^\pm(m, n||p - q| > k)$. The trend of visibility as a function of k is reported on Figure 7-(a). We observe that, increasing the value of k, hence detecting a higher unbalancement between $\vec{\pi}_+$ and $\vec{\pi}_-$ photons on mode $\mathbf{k_d}$, we obtain an higher visibility of the state on mode $\mathbf{k_c}$.

C. Analysis of the Macro-state $|\Phi^R\rangle$

Let us consider the case in which the state $|\Phi^R\rangle$ impinges on the UBS:

$$|\Phi^R\rangle = \frac{1}{C^2} \sum_{ij}^\infty \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^i \frac{\sqrt{(2j)!}}{j!} \frac{\sqrt{(2i+1)!}}{i!}$$
$$\times |(2i+1)R, (2j)L\rangle_b = \qquad (7)$$
$$= \frac{1}{C^2} \sum_{ij}^\infty \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^i \frac{(\mathbf{b_R^\dagger})^{2i+1}}{j!} \frac{(\mathbf{b_L^\dagger})^{2j}}{i!} |0\rangle$$

After the UBS the state can be written as:

$$|\Phi^R\rangle^{out} = \frac{1}{C^2} \sum_{ij}^\infty \sum_k^{2i+1} \sum_l^{2j} \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^i \frac{1}{j!}\frac{1}{i!} \frac{\sqrt{\tau}^{-k+l}}{\sqrt{k!}}$$
$$\frac{(2i+1)!(2j)!}{\sqrt{(2i+1-k)!(2j-l)!}} \frac{(\imath\sqrt{1-\tau})^{2i+1+2j-k-l}}{\sqrt{l!}}$$
$$|kR, lL\rangle_c|(2i+1-k)R, (2j-l)L\rangle_d$$
$$(8)$$

The state on mode $\mathbf{k_d}$ is then measured in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. The state $|(2i+1-k)R, (2j-l)L\rangle_d$ can then be rewritten as:

$$|(2i+1-k)R, (2j-l)L\rangle = \sum_r^{2i+1-k} \sum_s^{2j-l} \frac{1}{\sqrt{2}^{2i+1+2j-k-l}}$$
$$\frac{1}{\sqrt{(2i+1-k)!}} \frac{1}{\sqrt{(2j-l)}} \binom{2i+1-k}{s} \binom{2j-l}{s} \times$$
$$\sqrt{(s+r)!(2i+1+2j-k-l-s-r)!}\,\imath^{2i+1-k-r}\imath^{2j-l-s}$$
$$|(r+s)+, (2i+1+2j-k-l-r-s)-\rangle_d$$
$$(9)$$

and the overall state reads:

$$|\Phi^R\rangle^{out} = \frac{1}{C^2} \sum_{ij}^\infty \sum_k^{2i+1} \sum_l^{2j} \sum_r^{2i+1-k} \sum_s^{2j-l} \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^i \frac{1}{i!}\frac{1}{j!} \frac{(2i+1)!(2j)!\imath^{2i+1-k-r}(-\imath)^{2j-l-s}}{\sqrt{(2i+1-k)!(2j-l)!k!l!}} \frac{\sqrt{\tau}^{-k+l}\sqrt{\imath(1-\tau)}^{2i+1+2j-k-l}}{\sqrt{2}^{2i+1+2j-k-l}}$$
$$\frac{\sqrt{(r+s)!}\sqrt{(2i+1+2j-k-l-r-s)!}}{\sqrt{(2i+1-k)!(2j-l)!}} \binom{2i+1-k}{r} \binom{2j-l}{s} |(r+s)+, (2i+1+2j-k-l-r-s)-\rangle_d|kR, lL\rangle_c \quad (10)$$

If the state on mode $\mathbf{k_d}$ is detected : $|(r+s)+, (2i+1+ \quad 2j-k-l-r-s)-\rangle_d = |p+, q-\rangle_d$, the state on mode $\mathbf{k_c}$

is:

$$|\Phi^R\rangle^{meas} = \frac{1}{C^2} \sum_{ij}^{\infty} \sum_{l}^{2j} \sum_{s}^{2j-l} \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^i \frac{1}{i!}\frac{1}{j!} \frac{(2i+1)!(2j)!}{\sqrt{(l+p+q-2j)!(2j-l)!}} \frac{\sqrt{\tau}^{-2i+2j+1-p-q}}{\sqrt{(2i+1+2j-l-p-q)!l!}}$$
$$\frac{\sqrt{p!q!}}{\sqrt{2}^{p+q}} \frac{\sqrt{\imath(1-\tau)}^{p+q}}{\sqrt{(l+p+q-2j)!(2j-l)!}} \binom{l+p+q-2j}{p-s} \binom{2j-l}{s} \imath^{l+q-2j}(-\imath)^{2j-l-s}|(2i+1+2j-l-p-q)R,lL\rangle_c$$

$$(11)$$

where the following conditions have to be satisfied:

$$\begin{aligned} p &> s \\ 2i+1+2j &> l+p+q \\ 2j &< l+p+q \end{aligned} \tag{12}$$

If the state (11) is measured in the polarization basis $\{\vec{\pi}_R, \vec{\pi}_L\}$ obtaining a state $|mR, nL\rangle_c$, the corresponding probability amplitude is:

$$\frac{1}{C^2} \sum_{j}^{\infty} \sum_{s}^{2j-n} \left(\frac{\imath\Gamma}{2}\right)^j \left(\frac{\imath\Gamma}{2}\right)^{(-2j+m+n+p+q-1)/2} \frac{(-\imath)^{2j-n-s}}{\sqrt{m!n!}}$$
$$\frac{1}{j!} \frac{1}{(\frac{-2j+m+n+p+q-1}{2})!} \frac{(-2j+p+q+m+n)!(2j)!}{\sqrt{(n+q-2j+s)!(p-s)!}}$$
$$\frac{\sqrt{p!q!}}{(2j-n-s)!s!} \sqrt{\tau}^{m+n}(\imath\sqrt{1-\tau})^{p+q} \frac{1}{\sqrt{2}^{p+q}} \imath^{n+p+q-2j-r}$$

$$(13)$$

and the probability of measuring the state $|mR, nL\rangle_c$ is given by:

$$P(m,n|p,q) = \frac{1}{C^4} \sum_{j=0}^{\infty} \sum_{s}^{2j-n} \sum_{i}^{\infty} \sum_{r}^{2i-n} \frac{1}{C^4} \left(\frac{\Gamma}{2}\right)^{m+n+p+q-1}$$
$$\frac{p!q!}{m!n!} \frac{(-1)^{r-s}}{2^{p+q}} \frac{\tau^{m+n}(1-\tau)^{p+q}}{(\frac{p+q+m+n-2j-1}{2})!j!} \frac{1}{(\frac{p+q+m+n-2i-1}{2})!i!}$$
$$\frac{(p+q+m+n-2j)!(2j)!}{(n+q+s-2j)!s!(2j-n-s)!(p-s)!} \times$$
$$\frac{(p+q+m+n-2i)!(2i)!}{(q+n+r-2i)!r!(2i-n-r)!(p-r)!}$$

$$(14)$$

The visibility of the macro-state reads:

$$V(k) = \frac{\sum_{m,n} \sum_{p,q} \left(P_{m,n}^{p,q\ R}(k) - P_{m,n}^{p,q\ L}(k)\right)}{\sum_{m,n} \sum_{p,q} \left(P_{m,n}^{p,q\ R}(k) + P_{m,n}^{p,q\ L}(k)\right)} \tag{15}$$

where $P_{m,n}^{p,q\ R,L}(k) = P^{R,L}(m,n||p-q| > k)$. Here, $P^R(m,n||p-q| > k)$ is the probability that, given the detection of the state $|p+, q-\rangle_d$ on mode $\mathbf{k_c}$, $m > n$ is obtained on mode $\mathbf{k_d}$ ($m(n)$, number of photons polarized $\vec{\pi}_R(\vec{\pi}_L)$). In this case the state $|\Phi^R\rangle$ is identified; conversely the state $|\Phi^L\rangle$ is detected even if the state $|\Phi^R\rangle$ impinged on the UBS.

We observe that the visibility of the state $|\Phi^R\rangle$ in the case in which a small portion of the overall state is measured on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ polarization basis, is a decreasing function of the threshold k ($|p-q| > k$). This trend is shown in figure 7-(c). The decreasing trend of visibility can be explained by considering that the measurements in the two polarization basis correspond to two non-commuting operators acting on the same initial state. Indeed, for asymptotically high values of the threshold $k \to \infty$, the measurement of the $\hat{\Pi}_i$ operators that describe the OF tends to the measurement of the pseudo-spin operators $\hat{\Sigma}_i$: i.e $\hat{\Sigma}_1 = |\Phi^+\rangle\langle\Phi^+| - |\Phi^-\rangle\langle\Phi^-|$ or $\hat{\Sigma}_2 = |\Phi^R\rangle\langle\Phi^R| - |\Phi^L\rangle\langle\Phi^L|$. More details on the relationship between the OF device and the pseudo-Pauli operator can be found in Ref.[26]. In view of this consideration, the measurement on the $\mathbf{k}_C$ mode corresponds to the measurement of the $\hat{\Sigma}_i$ operators. The information gained on this mode about one of the two pseudo-spin operator acting on the macro qubit does not allow to gain information about orthogonal pseudo-spin operator. As a further remark, let us stress that this feature of the OF measurement is related to the filtering of different regions of the Fock space depending on the analyzed basis. The portion of the state that survives the action of the OF is indeed different if measured on the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis or in the $\{\vec{\pi}_R, \vec{\pi}_L\}$ one and is shown in figure 8.

IV. PROBABILISTIC TRANSMITTED STATE IDENTIFICATION

In the previous sections we have shown how the visibility of the macro qubit obtained by a pure dichotomic measurement can be modified if a small portion of the beam is identified by a probabilistic measurement strategy.

This section addresses the trend of the macro-states visibility when the field is split in two equal parts by
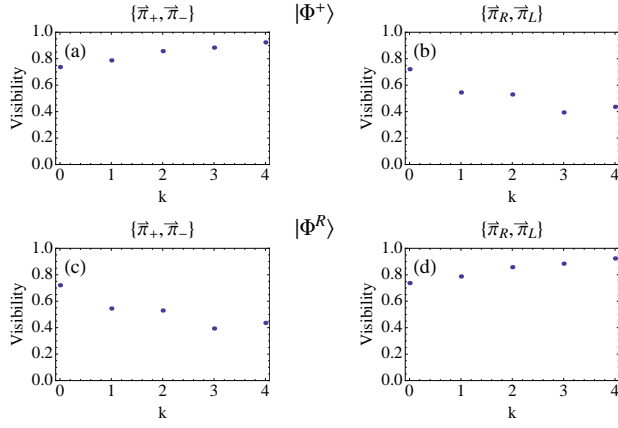
FIG. 7: (Color online) (a)-(b)Trend of the visibility of the state $|\Phi^+\rangle$ measured in the basis $\{\vec{\pi}_+, \vec{\pi}_-\}$ and $\{\vec{\pi}_R, \vec{\pi}_L\}$ respectively as a function of the threshold k. (c)-(d) Trend of the visibility of the state $|\Phi^R\rangle$ measured in the basis $\{\vec{\pi}_+, \vec{\pi}_-\}$ and $\{\vec{\pi}_R, \vec{\pi}_L\}$ respectively as a function of the threshold k. The numerical results have been obtained for the value of the gain parameter $g = 1.1$.
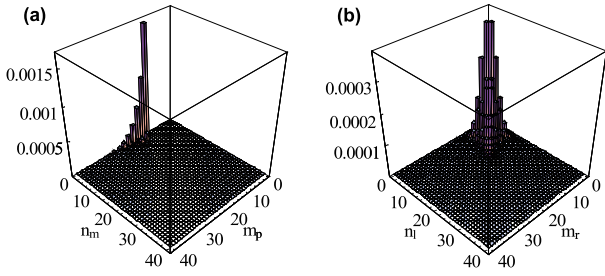

FIG. 8: (Color online) Selected region for the $|\Phi^+\rangle$ state after the measurement with an OF in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. (a) Photon number distribution in the $\{\vec{\pi}_+, \vec{\pi}_-\}$ basis. (b) Photon number distribution in the $\{\vec{\pi}_R, \vec{\pi}_L\}$ basis. In both cases $k = 10$ and $g = 1.2$.

a 0.5/0.5 beam-splitter and both the reflected and the transmitted states are detected through the OF device. In this case the measurement schemes are shown in figures 2-(III) and 9: the OF technique is applied in order to extract the maximum information available from the two measured states.

We consider the case in which the portion on the reflected mode is analyzed in the polarization basis orthogonal to the codification one. In figure 10 is reported the trend of visibility as a function of the threshold h on the transmitted mode, and k on the reflected one. The two polarization analysis basis are chosen to be mutually unbiased. It can be seen that for equal values of the two thresholds $h = k$ the visibility reaches a value around 0.64, the same obtained through a pure dichotomic measurement, without any pre-selection procedure on the macro-state. In figure 11 is reported the trend of the visibility as a function of the threshold on the reflected mode,

keeping fixed the value of the threshold on the transmitted one. We can see that the visibility of the transmitted state decreases when the threshold on the reflected mode increases. If the threshold on the transmitted mode is greater than the one on the reflected mode, the visibility results to be higher than 0.64, as expected by the action of the OF, which allows a better discrimination of the macro-state, measured in the codification polarization basis. Otherwise it can be seen how, decreasing the threshold h below the threshold k, the visibility decreases below the "no filtering value".
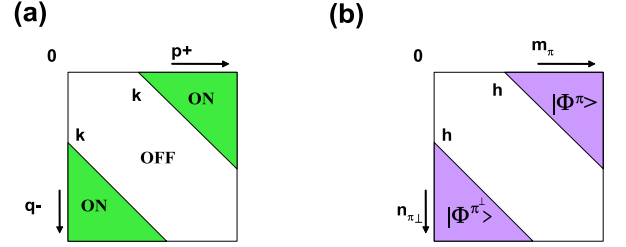

FIG. 9: (Color online)(a) Conditional activation of the shutter: if the OF acting on the reflected mode measures the state on the green regions, the shutter, on the transmitted mode, is conditionally activated. The green regions correspond to the state for which the signals belonging to orthogonal polarizations are unbalanced over a certain threshold k, i.e. $|p-q| \geq k$. (b) Corresponding to the ON region on the reflected mode, the transmitted mode is identified by a probabilistic measurement in the $\{\pi, \pi_\perp\}$ basis. The identification condition is $|m-n| \geq h$.
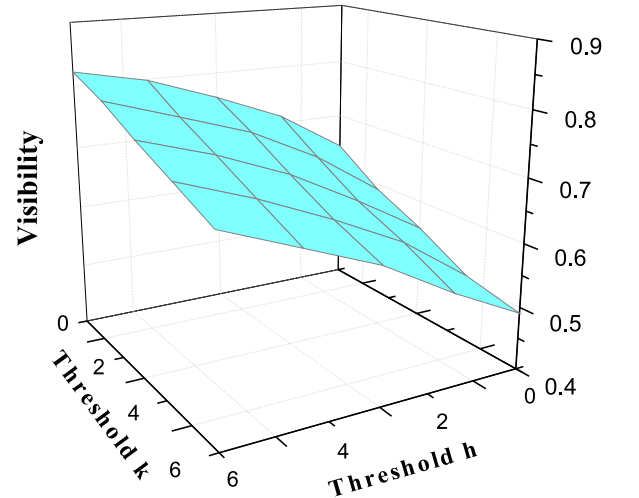

FIG. 10: (Color online) Trend of the visibility of the state $|\Phi^R\rangle$ for different values of the threshold h on the transmitted mode and of the threshold k on the reflected one. The numerical result has been obtained for a value of the parameter $g = 1.2$.

From the analysis performed in this paper we can conclude that the macro states are not suitable for quantum

cryptography. The action on a portion of the state can indeed be seen as an eavesdropping attack. If the state is measured in the codification basis, the visibility of the final state results to increase as shown in figures 7 (a)-(d). This means that the conclusive results for the eavesdropper would coincide with the conclusive results for the receiver, and the eavesdropper can gain information on the macrostates without introducing noise. Otherwise if the state is measured by the eavesdropper in the wrong basis, the visibility at the receiver is not affected if the state is measured above a certain filtering threshold. According to these considerations, an eavesdropper could then develop a strategy in which he measures its part of the transmitted state in two bases. With this approach he could gain information on the transmitted signal by considering only the measurement outcome in the right basis, and only a small amount of noise is introduced by keeping the filtering thresholds above a certain value. Related to the security of the macro-states is the possibility of performing a non-locality tests upon them. As a final remark for this section, we remind that the adoption of the OF device at the measurement stage is not suitable for a non-locality test, since the filtered portion of the state is *dependent* on the measurement basis [20]. We will then address the non-locality task in the following section.
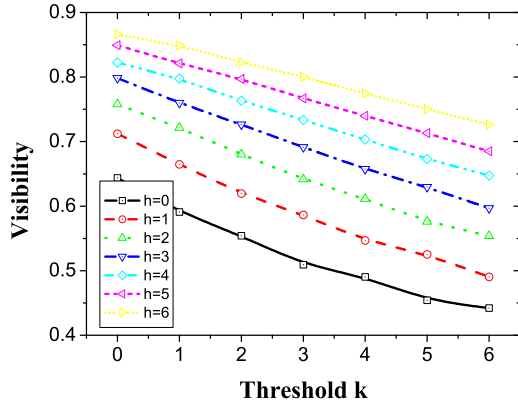

FIG. 11: (Color online) Trend of the macro-state visibility as a function of the threshold k on the reflected mode, fixed the threshold h on the transmitted one.

V. PRE-SELECTION FOR ENTANGLEMENT AND NON-LOCALITY TESTS

In this section we shall investigate a pre-selection scheme based on a conditional operation driven by the measurement of a portion of the multiphoton state in two different polarization basis. The setup of this pre-selection scheme is reported in fig.2-IV. A small portion of the generated multiphoton state is reflected by an unbalanced beam-splitter of transmittivity $T = 0.9$ and subsequently split by a 50/50 beam-splitter in two equal parts. One of the two parts is measured in an equatorial $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ basis by two photomultipliers, and the photocurrents $\{I_\beta, I_{\beta_\perp}\}$ are analyzed by an OF device [Fig.5]. The other part undergoes the same measurement process in a different equatorial basis $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$.

When the threshold condition $|I_\pi - I_{\pi_\perp}| > k$ [Fig.5] is realized in both branches, measured respectively in the polarization basis $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$, a TTL electronic signal is sent to conditionally activate the optical shutter placed in the optical path of the remaining part of the multiphoton state. Then, the field is analyzed at the measurement stage with the dichotomic strategy discussed in the previous paragraphs. For this pre-selection method, the relevant parameter is the angle ϕ between the two bases $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$ in which the small portion of the beam is analyzed. The angle ϕ is defined according to the relations between the two polarization bases:

$$\vec{\pi}_{\beta'} = e^{\imath\frac{\phi}{2}}\left[\cos\left(\frac{\phi}{2}\right)\vec{\pi}_\beta - \imath\sin\left(\frac{\phi}{2}\right)\vec{\pi}_{\beta_\perp}\right] \quad (16)$$

$$\vec{\pi}_{\beta'_\perp} = e^{\imath\frac{\phi}{2}}\left[-\imath\sin\left(\frac{\phi}{2}\right)\vec{\pi}_\beta + \cos\left(\frac{\phi}{2}\right)\vec{\pi}_{\beta_\perp}\right] \quad (17)$$

Let us begin by analyzing the trend of the visibility of the fringe pattern obtained by varying the equatorial polarization $\vec{\pi}_\alpha$ of the injected single-photon state in the amplifier. More specifically, we analyze how the visibility changes as a function of the angle ϕ between the two bases of the pre-selection branch. In Fig.12 we show the numerical results obtained by calculating the visibility according to the standard definition $V = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$. In this case, the visibility is evaluated according to the following expression:

$$V(k) = \frac{\sum_{m>n} P_{\overline{\alpha}}\left[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\right] - \sum_{m<n} P_{\overline{\alpha}}\left[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\right]}{\sum_{m>n} P_{\overline{\alpha}}\left[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\right] + \sum_{m<n} P_{\overline{\alpha}}\left[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\right]}$$
$$(18)$$

Here $P_{\overline{\alpha}}\left[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\right]$ is the photon-number distribution of the state $|\Phi^{\overline{\alpha}}\rangle$ after the pre-selection stage. More specifically, the value of $\overline{\alpha}$ is chosen in order to maximize the contribution of the $\sum_{m>n}$ term

and minimize the contribution of the $\sum_{m<n}$ term:

$$I_{max} = \sum_{m>n} P_{\overline{\alpha}}\Big[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\Big] \tag{19}$$

$$I_{min} = \sum_{m<n} P_{\overline{\alpha}}\Big[m,n\big|(|I_\beta - I_{\beta_\perp}| > k) \cap (|I_{\beta'} - I_{\beta'_\perp}| > k)\Big] \tag{20}$$

Eq.(18) then coincides with the usual definition of visibility. We note that the visibility is higher for smaller angles ϕ, since in that case a strong projection of the state is performed in two close bases. This condition is equivalent to the scheme of Fig.2-(II), where the OF measurement performed in one basis allows to obtain a better discrimination of the detected state only in the polarization basis of the pre-selection measurement [Fig.7 (a)-(b)]. When ϕ is high, a lower visibility can be achieved since the projection of the macrostate occurs in two distant bases. In this case, the increasing effect of the pre-selection in one basis on the visibility is in contrast with the decreasing effect of the pre-selection in the other basis, as shown in Sec.III.

We conclude this section by discussing the feasibility of a non-locality test by exploiting the proposed pre-selection method. We consider the case of a CHSH inequality [27]. Let us briefly summarize in the light of a local hidden variable (LHV) theory the content of Bell's inequalities for a set of dichotomic observables. Consider a quantum state described by the density matrix $\hat{\rho}$ defined in the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$. Define $\hat{O}_a^i$ the positive operator acting on subspace $\mathcal{H}_1$, and the probability of finding the value i after the measurement a as given by $\text{Tr}\Big[\hat{\rho}(\hat{O}_a^i \otimes \hat{I})\Big]$. The same relation holds for the positive operator $\hat{O}_b^j$ acting on subspace $\mathcal{H}_2$.

The existence of a LHV model implies that the expectation values of the a and b observables are predetermined by the value of the parameter λ: $\{X_a(\lambda), X_{a'}(\lambda), X_b(\lambda), X_{b'}(\lambda)\}$, hence the product $a \cdot b$ is equal to $X_a(\lambda)X_b(\lambda)$. For a fixed value of λ the variables X_n with $n = \{a, b, a', b'\}$ take the values $-1, 1$ and satisfy the CHSH inequality:

$$X_a(\lambda)X_b(\lambda) + X_a(\lambda)X_{b'}(\lambda) + X_{a'}(\lambda)X_b(\lambda) - X_{a'}(\lambda)X_{b'}(\lambda) \le 2 \tag{21}$$

The same inequality holds by integrating this equation on the space of the hidden variable (λ):

$$\int_\Omega d\mathbb{P}(\lambda)X_a(\lambda)X_b(\lambda) + \int_\Omega d\mathbb{P}(\lambda)X_a(\lambda)X_{b'}(\lambda) +$$
$$\int_\Omega d\mathbb{P}(\lambda)X_{a'}(\lambda)X_b(\lambda) - \int_\Omega d\mathbb{P}(\lambda)X_{a'}(\lambda)X_{b'}(\lambda) \le 2 \tag{22}$$

where $\mathbb{P}(\lambda)$ is the measure of the λ probability space. If there is a local hidden variables model for quantum
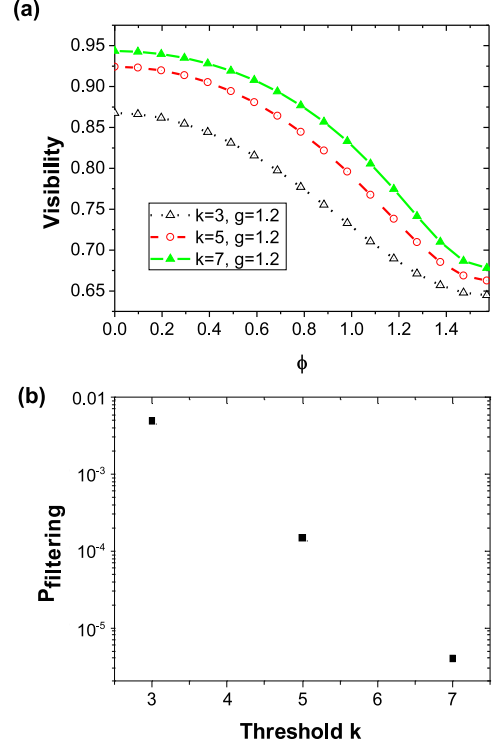

FIG. 12: (Color online) (a) Trend of the visibility for the double-filtering technique as a function of the angle ϕ between the two polarization bases $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$ of the pre-selection measurement. Square black points correspond to $k = 3$, circular red points to $k = 5$ and triangular green points to $k = 7$. (b) Filtering probability of the scheme as a function of the threshold k at the pre-selection measurement stage. All graphs correspond to $g = 1.2$.

measurement taking values $[-1, +1]$, then the following inequality must be satisfied:

$$S_{CHSH} = E^\rho(a,b) + E^\rho(a,b') + E^\rho(a',b) - E^\rho(a',b') \le 2 \tag{23}$$

where $E^\rho(a,b) = \int_\Omega X_a(\lambda)X_b(\lambda)d\mathbb{P}(\lambda)$. The violation of (23) proves that a LHV variables model for the considered experiment is impossible.

We consider the case in which the angle ϕ between the two bases $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$ is set at $\phi = \pi/4$. This choice is motivated by the following considerations. On one side, low values of ϕ would lead to a micro-macro state possessing strong correlations only in one polar-

ization basis, thus not allowing to violate a Bell's inequality. On the other side, high values of ϕ does not allow to obtain the necessary enhancement in the correlations of the micro-macro system to violate a Bell's inequality. The obtained fringe patterns for the chosen case are reported in Fig.13 and corresponds to the following conditions. The $(+1)$ outcome of the dichotomic measurement is recorded as a function of the polarization $\vec{\pi}_\alpha$ of the injected single photon state. In particular, the two chosen equatorial polarization bases $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ and $\{\vec{\pi}_{\beta'}, \vec{\pi}_{\beta'_\perp}\}$ corresponds to $\beta = 0$ and $\beta' = \frac{\pi}{4}$. We then analyzed three different choices for the threshold k at the pre-selection stage. When the threshold k is set to 0, the fringe pattern corresponding to the two basis $\beta = 0$ and $\beta = \frac{\pi}{4}$ are mutually shifted of an angle $\frac{\pi}{4}$, since no filtering and no pre-selection is performed on the state. When the threshold k is increased, the mutual shift between the fringe pattern is progressively reduced and cancelled, since a strong filtering of the state is performed. In particular, the maximum of both the fringe pattern in the $\beta = 0$ and $\beta = \frac{\pi}{4}$ bases is obtained for the $|\Phi^{\overline{\alpha}}\rangle$ state with $\overline{\alpha} = \frac{\pi}{8}$. This means that this pre-selection strategy for sufficiently high value of k enhances the correlations in the micro-macro system in a specific polarization basis and suppresses the correlations in the other bases. For this reason, the proposed strategy does not allow to observe the violation of a Bell's inequality in the micro-macro system here analyzed. The enhanced value of the visibility could nevertheless be employed in quantum lithography and quantum metrology schemes, in which high visibility correlations pattern and high photon number regimes are required. Recently it has indeed been shown how the amplification process of a single photon probe can beat the detrimental effect of losses which happen in the transmission and detection stages [28]. Such a scheme for non invasive quantum metrology could benefits from the presented filtering procedures in order to improve the visibility value of the interference fringe pattern.

VI. CONCLUSIONS

In this paper we have analyzed the properties of the macro states obtained by a quantum injected amplification process, by addressing the behavior of the distinguishability between orthogonal macro-states when a filtering process is applied over a portion of them. More specifically, we analyzed theoretically in details several schemes for the realization of conditional measurement-induced operations. All these strategies are aimed at the manipulation and filtering of the macro-states for their applications in different contexts, such as the realization of a non-locality test or quantum communication.

We have identified a strategy, based on the ID device, able to minimize the effects of the noise due to the vac-

uum injection into the amplifier. The ID based filtering procedure is independent on the analysis basis and selects the same portion of the state when the measurement is performed in any equatorial polarization basis.

A different filtering procedure, based on the OF device, has been deeply studied: it turned out that when a small portion of the state is analyzed through the OF, the visibility of the overall state, relative to a dichotomic measurement, is affected in a different way depending on the polarization basis in which the small portion has been measured. If the polarization basis is the same of the macro qubit codification, the final visibility increases with the increase of the filtering threshold, otherwise it decreases. This behavior is related with the impossibility of measuring non commuting operators on the same quantum state, as explained in Section III.

We have further addressed in Sec.IV the trend of the macro state visibility when an OF discrimination system is used even at the transmitted state detection stage. In this case, the two OF apparatuses in both transmitted and reflected branches play an opposing role in increasing or decreasing the visibility of the fringe-pattern obtained in a micro-macro configuration. Such analysis shows that the macro-states generated by optical parametric amplification of a single-photon state are not suitable for quantum cryptography, since they are not robust under an eavesdropping attack. Furthermore, we showed that this discrimination method is not suitable for a micro-macro non-locality test since it performs a base-dependent filtering of the detected state.

Finally, in Sec.V we addressed a pre-selection scheme for the realization of a Bell's inequality test which do not suffer the same detection loopholes of the one based on post-selection strategies [20]. The proposed method, based on the measurement of the reflected part of the wave-function in two different bases, does not allow to violate a Bell's inequality, since it induces the collapse of the correlations present in the macro-states in only a single polarization basis.

Several open points remain to be investigated. The measurement-induced operations analyzed in this paper are all based on dichotomic detection schemes. Other approaches, such as the ones based on continuous variables measurements or on the processes of coherent photon-addition and photon-subtraction, can lead to a different manipulation of the QIOPA multiphoton states. Systems with different properties from the one analyzed in this paper could be obtained with these methods.
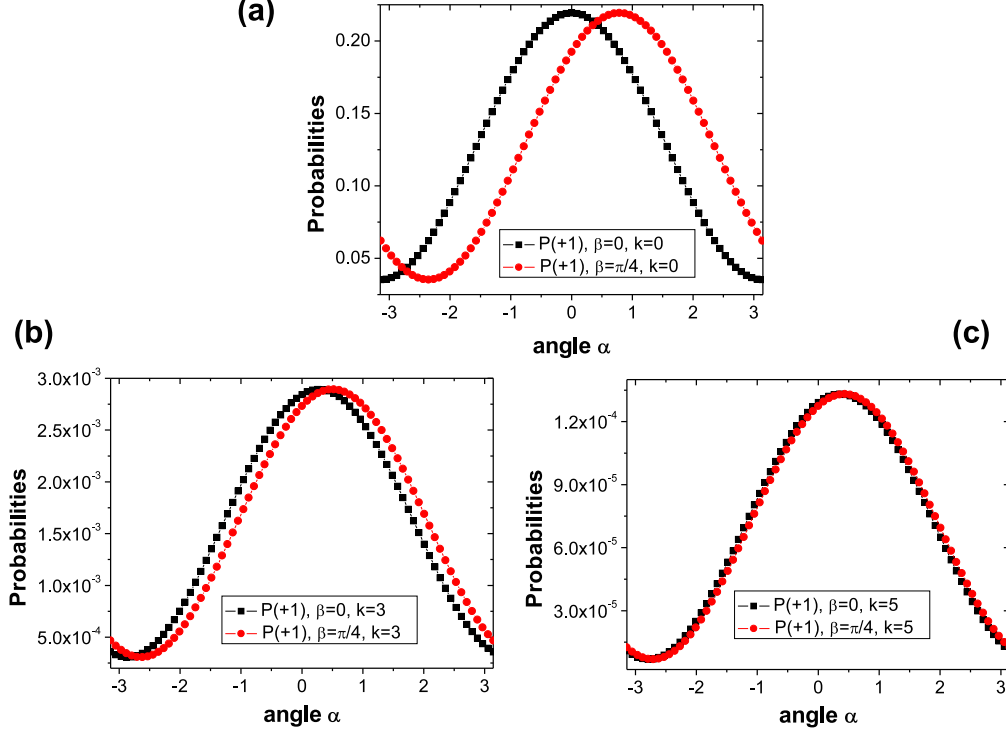
FIG. 13: (Color online) Fringe pattern as a function of the angle α of the polarization basis at the single-photon site. The angle ϕ between the two bases of the pre-selection stage is set at $\phi = \pi/4$, while $g = 1.2$. (a) Threshold $k = 0$. (b) Threshold $k = 3$. (c) Threshold $k = 5$. Square black points: fringe patterns obtained by recording the $(+1)$ outcome at the measurement stage, where the measurement basis $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ is set at $\beta = 0$. Circle red points: fringe patterns obtained by recording the $(+1)$ outcome at the measurement stage, where the measurement basis $\{\vec{\pi}_\beta, \vec{\pi}_{\beta_\perp}\}$ is set at $\beta = \frac{\pi}{4}$.

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